

# Optimized Inductive Power Transfer Using Randomized Method

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**Abstract:** An analysis of the transferred power of an inductive link circuit with different network configurations of capacitors connected to primary and secondary coils. The performance for both objective functions was observed using four capacitor, two capacitors connected to the input coil and two capacitors connected to the output coil. However, the output in this circuit configuration for both efficiency and output power are very complex and a numerical method had to be applied to calculate the capacitors values. Since an in-depth search would be long therefore, some simplifications were assumed to reduce the search space and the time. Therefore, an algorithm based on a randomized method is developed and successfully applied. The results for both efficiency and output power of four capacitors configuration are compared with other approaches, such as the single and two capacitors compensation. Finally, a basic prototype was built and the theoretical results were validated. Both simulated and experimental results of the four capacitor configuration showed a significant improvement on the efficiency and output power of the inductive link.

**Key words:** electromagnetic coupling, optimization.

## I. INTRODUCTION

Electronic devices have seen substantial lowering in energy consumption with the integrated circuit technology advances. Hence, the batteries did not become less important or dispensable, but they turned into one of the heaviest components in modern portable electronic devices [1]. For instance, in implantable devices, the small size of batteries is essential. In such systems, the battery has a direct effect on the user's life and some malfunction is a

serious threat to the patient's health [2]. In addition, as an alternative to batteries, the use of a power cord becomes a problem due to reliability and maintenance [3]. One possible replacement to batteries or power cords is powering through magnetically coupled coils. This alternative was already broadly employed in many different applications, where a contactless power system is a necessity, such as biomedical devices [2], [4], [5] and instrumentation systems [6], among others [7]–[9]. The magnetically coupled system is usually represented by two inductances,  $L_1$  (primary side) and  $L_2$  (secondary side), and a low mutual inductance  $M$  [10]. [10] Present a compensated inductive link with only one capacitor. In addition, a system controls the frequency of the oscillator source by means of a wireless communication to adjust the maximum power on the load. Another technique, also based on SP compensation, is proposed in [13]. Kiani and Ghovanloo [14] presented an analysis between reflected load theory and coupled-mode theory and a system compensation based on an SP configuration. Although many authors tackled the subject by different methods, the complexity of the equations is a limiting factor in the analysis of a circuit compensated with more than two capacitors. [8], [11], and [12] presented four circuit topologies to power transfer through inductive coupling that make use of only two capacitors. The topologies are: compensation by a capacitor in series with the primary and secondary coils (SS), a capacitor in series with the primary and another capacitor in parallel with the secondary coil (SP), a capacitor in parallel with the primary and another capacitor in parallel with the secondary coil (PP), and a capacitor in

parallel to primary coil with another capacitor in series with the secondary coil (PS). The term compensation is about the utilization of a capacitor in series or parallel with the respective coil to reach resonance, or the cancelation of the inductive reactance. Brusamarello *et al.*

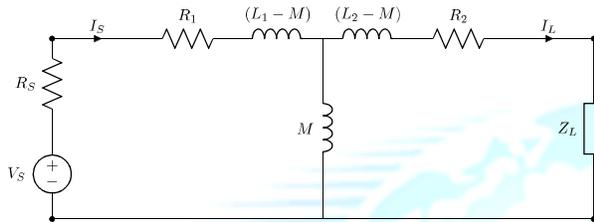


Fig. 1. Transformer model equivalent circuit

This paper analysis of an inductive link connected to two capacitor on both input as well as output coils [15].

The flow of paper is as section I discusses about the introduction with survey. Section II develops and presents the system of equations necessary to implement the full four capacitor circuit's compensations for both objective functions: output power and efficiency. The results are presented and compared with other useful configurations in Section III. Then, the conclusion, discussion, and future works are exposed in Section IV.

## II. METHODOLOGY

The basic wireless power transfer system composed by two coils,  $L_1$ (primary) and  $L_2$ (secondary), the equivalent losses in the coil,  $R_1$ (primary) and  $R_2$ (secondary), and mutual inductance  $M = k\sqrt{L_1 L_2}$  can be represented by an equivalent T circuit of a transformer model (Fig. 1), where  $V_s$  is a sinusoidal source with an internal resistance  $R_s$  and  $Z_L$  is the load. One should notice the behavior of the circuit from Fig. 1 when the coupling coefficient  $k$  is very small. In these cases, analyzing the primary side only, the inductance  $M$  can be simplified to a short circuit because  $j\omega M \ll j\omega L_1$  and  $j\omega M \ll j\omega L_2$ . Therefore, the primary is not influenced by the secondary load.

Considering a steady state  $R_s = 0$  and the load  $Z_L = R_L$  (For analysis simplifications), the total efficiency ( $\beta$ ) of the circuit of Fig. 1 and the output power on  $R_L$  are given by

$$\beta = \frac{1}{1 + \frac{L_2 R_1}{k^2 L_1 R_L} + \frac{a}{b} + \frac{R_2}{R_L}} \quad (1)$$

Where  $a = R_L(R_2 + R_L)^2$  &  $b = k^2 L_1 L_2 R_L \omega^2$

$$P_{out} = \frac{0.5}{C_1 + C_2 + C_3} \quad (2)$$

It is clear, from (1), that the only way to improve the total efficiency ( $\beta$ ) of the basic wireless transfer system shown in Fig. 1 without a circuit modification is to increase the frequency  $\omega$ . When  $k$ ,  $L_1$ ,  $L_2$ ,  $R_1$ ,  $R_2$ , and  $R_L$  are constants, the maximum theoretical efficiency will be reached when  $\omega \rightarrow \infty$  (3). However, the power on the load goes to zero in this situation and the resulting circuit is useless, as can be observed with (2).

$$\beta_m = \frac{1}{1 + \frac{L_2 R_1}{k^2 L_1 R_L} + \frac{R_2}{R_L}} \quad (3)$$

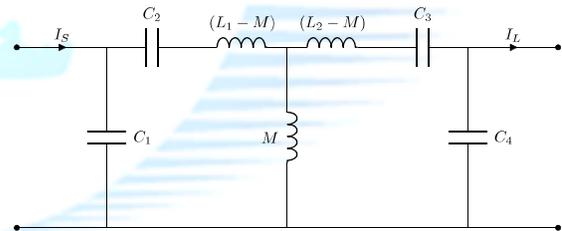


Fig. 2. Transformer model equivalent circuit.

In addition to the total efficiency ( $\beta$ ), the efficiency of the primary side and the secondary side of the circuit.

A very basic and intuitive analysis is supposing the circuit of Fig. 1 composed by ideal inductors (without loss resistances  $R_1$  and  $R_2$ ) and with a resistive load. In this particular case, four ideal capacitors would cancel all the inductances (Fig. 2). This would lead the load directly connected to the input voltage source by way of the resonance.

At resonance, the capacitance  $c_2$  cancels the inductance  $(L_1 - M)$ ,  $c_3$  cancels the inductance  $(L_2 - M)$ , and  $c_1$  and  $c_4$  cancel the inductance  $M$ .

$$c1 + c4 = \frac{1}{M\omega^2}$$

$$c2 = \frac{1}{(L1-M)\omega^2}$$

$$c3 = \frac{1}{(L2-M)\omega^2}$$

However, when lossy resistors are added to the voltage source ( $R_s$ ), capacitors ( $R_{c1}$  to  $R_{c4}$ ), and inductors ( $R_1$  and  $R_2$ ), the analytical equations for the objective functions (efficiency or output power) become very complicated. In this case, the optimization model is highly nonlinear and has more than one local minimum, as shown in Fig. 3, which represents the output power as a function of only two capacitors with the remaining variables held constant. The inclusion of additional variables and simultaneously accounting for the discrete nature the capacitor values makes the optimization model more complex and non-convex. Therefore, the use of classical methods of optimization becomes unattractive and a numerical method is suitable to find the capacitors values that improve power output and efficiency of the inductive link. The simplest method would be an exhaustive search [17], but computationally very costly. Thus, we adopted a simple approach based on a randomized method. Taking samples randomly from the search space would lead to the optimal point if there are an infinite number of trials [18]. With a finite number of trials, one can get as near as necessary to the

optimal point, depending only on the computational power available.

Although there may be many solutions to the optimization problem, its practical implementation can only be performed to a limited set of capacitors. This set of capacitors (disregarding associations) is composed by commercial values of capacitances. Indeed, this is a constraint of the optimization problem. Thus, the search space of possible values of capacitance was limited to 216 commercial

components (based on the IEC 60063 E24 series [19] multiplied by  $10^{-12}$  up to  $10^{-4}$ ), which give a total of 2176782336 possibilities. The basic idea is to randomly pick up four capacitors from the 216 possible values (a discrete and uniform distribution) and calculate the objective function (output power on the load and efficiency). This procedure is repeated a reasonable number of times ( $10^7$  trials in this paper or 0.46% of the total search space). Since the problem is defined by  $N_c^C$  (where  $N_c$  is the total number of different capacitances and  $C$  is the number of capacitors used in the circuit), it is clear that the circuit compensated by two capacitors can be calculated with all the possible values (46656) with a low computational cost.

As a simplification to the problem, all series lossy resistances are considered equal  $R_c = 0.1$  (based on the experimental setup).

The basic algorithm employed to find the best set of four capacitors is as follows:

- 1) initialize best  $\beta = 0$  and best  $P_o = 0$ ;
- 2) store  $n$  constants in a vector (in this paper represents 216 different values of capacitances);
- 3) randomly select a number from one to  $n$  ( $n = 216$ ) independently and uniformly distributed. These numbers are used as indexes of the array of capacitances. This procedure is repeated to generate the values for  $C_1$ ,  $C_2$ ,  $C_3$ , and  $C_4$ ;
- 4) calculate  $\beta$  and  $P_{out}$ ;
- 5) test if  $\beta$  and  $P_{out}$  are better than best  $\beta$  and best  $P_{out}$ , respectively. If one test is true, store the capacitors values and update that best value variable;
- 6) go to step 3 until the limit number of iterations is reached.



Fig. 4. Prototype variable coupling measurements.

### III. RESULTS

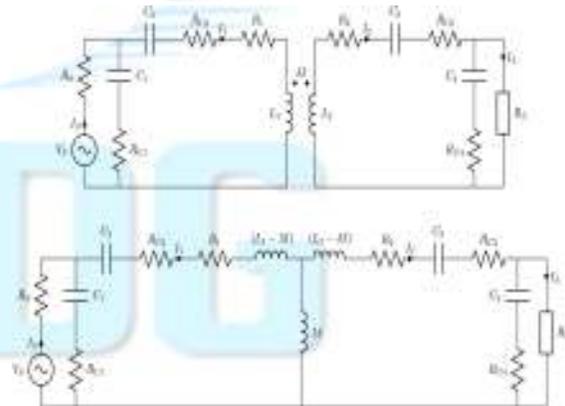
This section presents the results of computed values and simulations based on the circuit parameters from the prototyped inductive link shown in Fig. 4. As described in the previous section, the solutions are discrete, arranged to a set of 216 possible values of capacitances. The parameters of the experimental circuit are:  $R_S = 0.1$ ,  $R_1 = 1.8$ ,  $R_2 = 2.28$ ,  $L_1 = 218.4 \mu\text{H}$ , and  $L_2 = 311.4 \mu\text{H}$ . All capacitors, even if not mentioned in the text, are assumed to have a series lossy resistance  $R_C = 0.1$ . The voltage source  $V_S$  is sinusoidal with  $V_{\text{peak}} = 5 \text{ V}$  and  $f = 50 \text{ kHz}$ . Again, the load was simplified with  $Z_L = R_L$ . The main objective of this section is to present a comparison of the efficiency and output power (power on the load) of the circuit from Fig. 1 against the results of the configurations with only one compensation capacitor: series with the input (1Cap-SI), parallel with the input (1Cap-PI), series with the output (1Cap-SO), and parallel with the output (1Cap-PO); with two compensation capacitors (2Cap-SS, 2Cap-SP, 2Cap-PS, and 2Cap-PP); and the ideal circuit compensated by four ideal capacitors and the full four capacitor compensation considering all lossy resistances. To reduce the amount of data, the circuits output are evaluated with only five resistive loads values  $R_L$  (6.8, 47, 270, 470, and  $1\text{k}$ ) and only four coupling coefficient  $k$  (0.004, 0.04, 0.46, and 0.88). These first two values of  $k$  represent weak coupling such as two loosely coupled coils. One should notice that both

In addition, the method proposed in this paper provides a simple way to accomplish a multi objective search (efficiency and output power) and find the Pareto frontier from the calculated points with a very little increase on computational costs.

In the following section, the results of the circuit with four capacitors compensation are presented and compared with other useful circuit configurations for efficiency ( $\beta$ ) and output power ( $P_{\text{out}}$ ).

solutions of circuits compensated by one and two capacitors were computed with exhaustive search (as described in the previous section). The compensation capacitors of the ideal circuit were calculated with and approximated with the E24 series. Finally, the solutions of the full compensated circuit considering all.

Fig. 5. Inductive link circuit and the equivalent T model with four capacitors configuration.



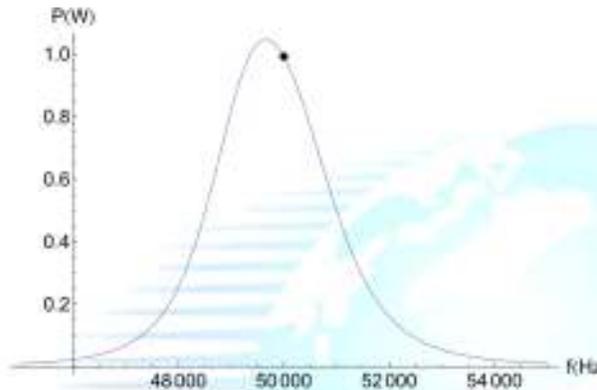


Fig. 6. Dependence of the power on the load  $P_{out}$  with  $R_L = 47 \Omega$  &  $k = 0.04$  with the circuit of Fig. 5 when frequency is varying.

The experimental circuit's parameters were measured: the adjusted coupling factor was  $k = 0.035$ , the output source's resistance was estimated at  $R_S = 0.2$ , and the sinusoidal voltage source set to 2.5 Vpp. In addition, the load resistances were measured and the variables: input and output voltages.

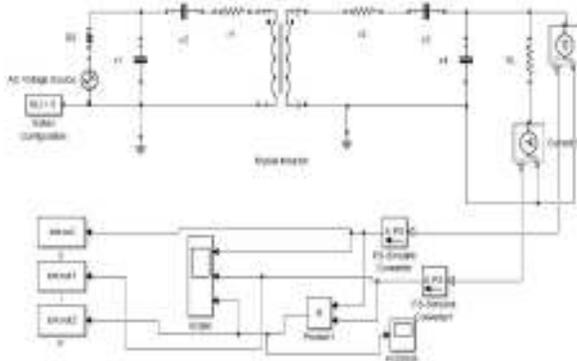
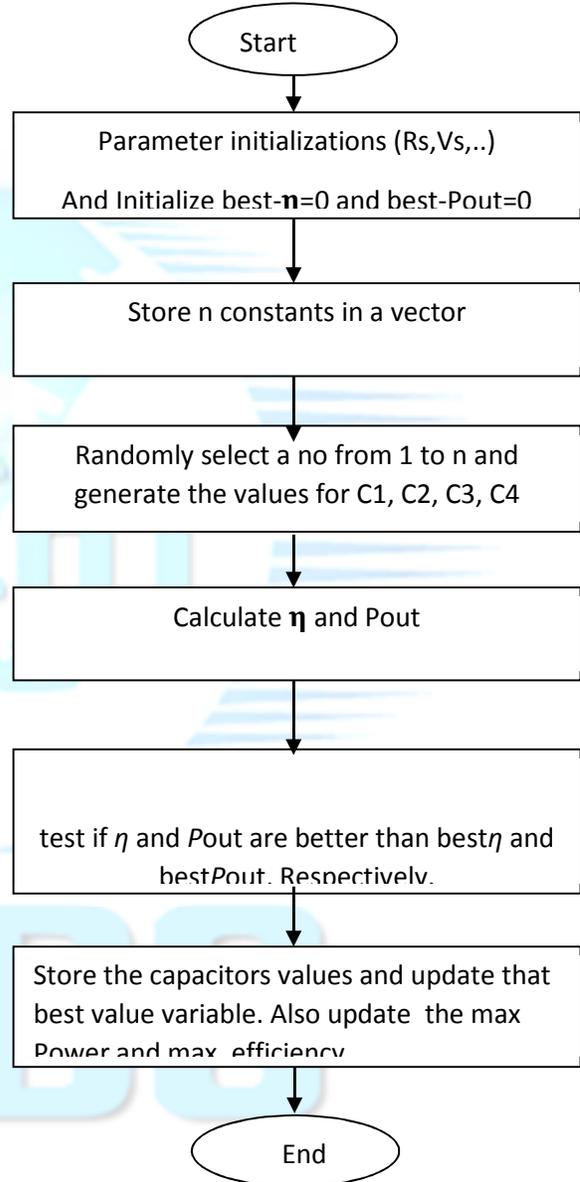


Fig. 7. Simulink model of 4 capacitor system.

$V_{IN}$  (Vp) and  $V_{OUT}$  (Vp), phase between  $V_{IN}$  and  $V_{OUT}$ ,  $\phi$  (rad) were monitored to compute the output power on the load and the efficiency of the inductive link. These parameters were also used to generate the simulation results. Table 2 shows both measured and simulated results for output power on the load and

efficiency of the inductive link prototype compensated with the capacitors of Table 1 with

Fig. 8: Flow chart of randomized method



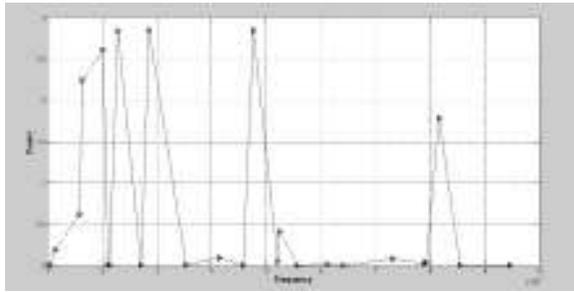


Fig. 9 Randomized power

By comparing the measured and simulated results presented in Table 2, one can observe that most of the cases match. This circuit was also simulated by SPICE presenting practically the same results. The small differences in results can be assigned to measurement errors of parameters or variables, as well as possible non-idealities of the ac source. Thus, we believe that the experimental values found in this paper serve to validate the presented theoretical

Table 1. Simulation results of 4 Capacitor configuration with different values of k, f & RL

k	RL (Ohms)	Power (Watts)		
		50k	650k	1000k
	Freq (Hz)			
0.04	6.8	1.3e-3	0.25e-6	1.2e-6
0.04	47	8e-3	1.2e-6	25e-6
0.04	1000	8.4e-5	0.7e-6	1.12e-6
0.46	6.8	.03	.6e-4	2.5e-5
0.46	47	1.447	2e-4	5.5e-5
0.46	1000	3.18	5e-4	20e-5
0.88	6.8	.165	9.8e-3	1.1e-3
0.88	47	.863	.015	3.5e-3
0.88	1000	3.2	.0165	3e-3

Table 2. Randomized Method results of no capacitor configuration

RL (Ω)	k	f (Hz)	P(W)	η(%)
10	0.04	10000	0.6528e-6	0.4448
50	0.04	50000	0.1305e-6	2.3647
500	0.04	100000	0.3259e-6	3.9441
10	0.46	10000	138.43e-06	34.262
50	0.46	50000	27.68e-06	73.667
500	0.46	100000	69.06e-06	84.127
10	0.88	10000	0.006174	59.173
50	0.88	50000	0.001236	88.432
500	0.88	100000	0.0030084	94.798

#### IV. CONCLUSION

This paper presents different configurations of circuit's compensations as means of improving the efficiency and the energy available to the load in power transmission systems that use inductively coupled coils. The performance of these

configurations was compared with a proposed and implemented four-capacitor compensation method.

Initially, we made a theoretical analysis of the efficiency and power delivered to a load by the inductive link featuring four capacitors and it became clear that in both cases, the analytical equations are very complicated. Furthermore, the values of the discrete components are natural constraints of experimental circuits, generating a discrete space of solutions. Thus, the capacitors values were computed using a search method based on a limited number of trials. The results of this paper represent the optimal values for both (not simultaneous) objective functions in  $10^7$  trials. The efficiency and output power from this circuit were computed with the application of the capacitor's values found by the algorithm in each trial. Thus, the best results for each load resistor with a set of capacitors were chosen and compared with the results of the inductive link compensated with other usual approaches, such as one and two capacitors connected on the input or output.

Table 3 Analysis based on results calculated from different techniques

PARAMETER	VALUE	VOLTAGE	POWER	EFFICIENCY
FREQUENCY	INCR.	DEC.	DEC.	INCR.
FREQUENCY	DEC.	INCR.	INCR.	DEC.
LOAD RESISTANCE	INCR.	INCR.	DEC.	DEC.

The compensation method presented in this paper was the only one to have a good performance, for both efficiency and output power, for all combinations of load  $R_L$  and tested coupling coefficients  $k$ . The results of power on the load using the inductive link with four capacitors compared with

the results of the same uncompensated inductive link ranged 7–124000 times higher. These results are even better when looking to low coupling coefficients, such as  $k = 0.004$  and  $k = 0.04$ . The power available on the load in this paper was always equal to or greater than the power on the load provided by any other configuration.

In addition, the inductive link compensated with four capacitors obtained a significative improvement of efficiency when compared with the results of the same inductive link with no capacitors and up to 130 times when compared with the results of compensated circuits with one and two capacitors. In addition, the secondary efficiency of the inductive link, a serious issue considering implantable devices, hits 75% with a coupling coefficient of only 0.004 in one specific case simulated with SPICE using the four-capacitor compensation.

The capacitor values determined by the described random method were compared with other usual optimization methods. The most of values are very close to the values obtained by classical optimization methods (reduced gradient). Such methods are especially sensitive to initialization on nonconvex problems. We also observed that the results have different sensibilities with the capacitors. For instance, the power on the load has weak dependence on  $C_1$  and efficiency has weak dependence on  $C_1$  and  $C_2$ . The power on the load and the efficiency of the circuit was also simulated by SPICE presenting practically the same results. Finally, the theoretical and simulated results were confronted and validated with measurements made with an inductive link prototype and most of the results match.

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